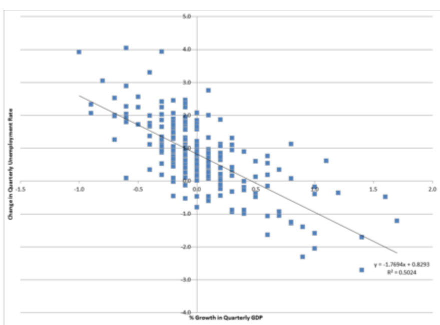
Econometrics/ Quantitative Economics

Econometrics is the application of statistical methods to economic data in order to give empirical content to economic relationships.[1] More precisely, it is "the quantitative analysis of actual economic phenomena based on the concurrent development of theory and observation, related by appropriate methods of inference".[2] An introductory economics textbook describes econometrics as allowing economists "to sift through mountains of data to extract simple relationships".[3] The first known use of the term "econometrics" (in cognate form) was by Polish economist Paweł Ciompa in 1910.[4] Jan Tinbergen is considered by many to be one of the founding fathers of econometrics.[5][6][7] Ragnar Frisch is credited with coining the term in the sense in which it is used today.[8]

A basic tool for econometrics is the multiple linear regression model.[9] Econometric theory uses statistical theory and mathematical statistics to evaluate and develop econometric methods.[10][11] Econometricians try to find estimators that have desirable statistical properties including unbiasedness, efficiency, and consistency. *Applied econometrics* uses theoretical econometrics and real-world data for assessing economic theories, developing econometric models, analysing economic history, and forecasting.

Basic models: linear regression

A basic tool for econometrics is the multiple linear regression model.[9] In modern econometrics, other statistical tools are frequently used, but linear regression is still the most frequently used starting point for an analysis.[9] Estimating a linear regression on two variables can be visualised as fitting a line through data points representing paired values of the independent and dependent variables.



Okun's law representing the relationship between GDP growth and the unemployment rate. The fitted line is found using regression analysis.

For example, consider Okun's law, which relates GDP growth to the unemployment rate. This relationship is represented in a linear regression where the change in unemployment rate ( )

is a function of an intercept ( ), a given value of GDP growth multiplied by a slope coefficient and an error term, :

The unknown parameters and can be estimated. Here is estimated to be −1.77 and is estimated to be 0.83. This means that if GDP growth increased by one percentage point, the unemployment rate would be predicted to drop by 1.77 points. The model could then be tested for statistical significance as to whether an increase in growth is associated with a decrease in the unemployment, as hypothesized. If the estimate of were not significantly different from 0, the test would fail to find evidence that changes in the growth rate and unemployment rate were related. The variance in a prediction of the dependent variable (unemployment) as a function of the independent variable (GDP growth) is given in polynomial least squares.

Theory[edit]

See also: Estimation theory

Econometric theory uses statistical theory and mathematical statistics to evaluate and develop econometric methods.[10][11] Econometricians try to find estimators that have desirable statistical properties including unbiasedness, efficiency, and consistency. An estimator is unbiased if its expected value is the true value of the parameter; it is consistent if it converges to the true value as the sample size gets larger, and it is efficient if the estimator has lower standard error than other unbiased estimators for a given sample size. Ordinary least squares (OLS) is often used for estimation since it provides the BLUE or "best linear unbiased estimator" (where "best" means most efficient, unbiased estimator) given the Gauss-Markov assumptions. When these assumptions are violated or other statistical properties are desired, other estimation techniques such as maximum likelihood estimation, generalized method of moments, or generalized least squares are used. Estimators that incorporate prior beliefs are advocated by those who favour Bayesian statistics over traditional, classical or "frequentist" approaches.

Methods[edit]

Main article: Methodology of econometrics

Applied econometrics uses theoretical econometrics and real-world data for assessing economic theories, developing econometric models, analysing economic history, and forecasting.[12]

Econometrics may use standard statistical models to study economic questions, but most often they are with observational data, rather than in controlled experiments.[13] In this, the design of observational studies in econometrics is similar to the design of studies in other observational disciplines, such as astronomy, epidemiology, sociology and political science. Analysis of data from an observational study is guided by the study protocol, although exploratory data analysis may be useful for generating new hypotheses.[14] Economics often analyses systems of equations and inequalities, such as supply and demand hypothesized to be in equilibrium. Consequently, the field of econometrics has developed methods for identification and estimation of simultaneous-equation models. These methods are analogous to methods used in other areas of science, such as the field of system identification in systems analysis and control theory. Such methods may allow researchers to estimate models and investigate their empirical consequences, without directly manipulating the system.

One of the fundamental statistical methods used by econometricians is regression analysis.[15] Regression methods are important in econometrics because economists typically cannot use controlled experiments. Econometricians often seek illuminating natural experiments in the absence of evidence from controlled experiments. Observational data may be subject to omitted-variable bias and a list of other problems that must be addressed using causal analysis of simultaneous-equation models.[16]

In addition to natural experiments, quasi-experimental methods have been used increasingly commonly by econometricians since the 1980s, in order to credibly identify causal effects.[17]

Example[edit]

A simple example of a relationship in econometrics from the field of labour economics is:

This example assumes that the natural logarithm of a person's wage is a linear function of the number of years of education that person has acquired. The parameter measures the increase in the natural log of the wage attributable to one more year of education. The term is a random variable representing all other factors that may have direct influence on wage. The econometric goal is to estimate the parameters, under specific assumptions about the random variable . For example, if is uncorrelated with years of education, then the equation can be estimated with ordinary least squares.

If the researcher could randomly assign people to different levels of education, the data set thus generated would allow estimation of the effect of changes in years of education on wages. In reality, those experiments cannot be conducted. Instead, the econometrician observes the years of education of and the wages paid to people who differ along many dimensions. Given this kind of data, the estimated coefficient on Years of Education in the equation above reflects both the effect of education on wages and the effect of other variables on wages, if those other variables were correlated with education. For example, people born in certain places may have higher wages and higher levels of education. Unless the econometrician controls for place of birth in the above equation, the effect of birthplace on wages may be falsely attributed to the effect of education on wages.

The most obvious way to control for birthplace is to include a measure of the effect of birthplace

in the equation above. Exclusion of birthplace, together with the assumption that is uncorrelated with education produces a misspecified model. Another technique is to include in the equation additional set of measured covariates which are not instrumental variables, yet

render identifiable.[18] An overview of econometric methods used to study this problem were provided by Card (1999).[19]

Journals[edit]

The main journals that publish work in econometrics are *Econometrica*, the *Journal of Econometrics*, the *Review of Economics and Statistics*, *Econometric Theory*, the *Journal of Applied Econometrics*, *Econometric Reviews*, the *Econometrics Journal*,[20] *Applied Econometrics and International Development*, and the *Journal of Business & Economic Statistics*.

Limitations and criticisms[edit]

See also: Criticisms of econometrics

Like other forms of statistical analysis, badly specified econometric models may show a spurious relationship where two variables are correlated but causally unrelated. In a study of the use of econometrics in major economics journals, McCloskey concluded that some economists report p-values (following the Fisherian tradition of tests of significance of point null-hypotheses) and neglect concerns of type II errors; some economists fail to report estimates of the size of effects (apart from statistical significance) and to discuss their economic importance. She also argues that some economists also fail to use economic reasoning for model selection, especially for deciding which variables to include in a regression.[21][22]

In some cases, economic variables cannot be experimentally manipulated as treatments randomly assigned to subjects.[23] In such cases, economists rely on observational studies, often using data sets with many strongly associated covariates, resulting in enormous numbers of models with similar explanatory ability but different covariates and regression estimates. Regarding the plurality of models compatible with observational data-sets, Edward Leamer urged that "professionals ... properly withhold belief until an inference can be shown to be adequately insensitive to the choice of assumptions".[23]

Macroeconomic model

A **macroeconomic model** is an analytical tool designed to describe the operation of the problems of economy of a country or a region. These models are usually designed to examine the comparative statics and dynamics of aggregate quantities such as the total amount of goods and services produced, total income earned, the level of employment of productive resources, and the level of prices.

Macroeconomic models may be logical, mathematical, and/or computational; the different types of macroeconomic models serve different purposes and have different advantages and disadvantages.[1] Macroeconomic models may be used to clarify and illustrate basic theoretical principles; they may be used to test, compare, and quantify different macroeconomic theories; they may be used to produce "what if" scenarios (usually to predict the effects of changes in monetary, fiscal, or other macroeconomic policies); and they may be used to generate economic forecasts. Thus, macroeconomic models are widely used in academia in teaching and research, and are also widely used by international organizations, national governments and larger corporations, as well as by economic consultants and think tanks.

Types

Simple theoretical models

Simple textbook descriptions of the macroeconomy involving a small number of equations or diagrams are often called 'models'. Examples include the IS-LM model and Mundell–Fleming model of Keynesian macroeconomics, and the Solow model of neoclassical growth theory. These models share several features. They are based on a few equations involving a few variables, which can often be explained with simple diagrams.[2] Many of these models are static, but some are dynamic, describing the economy over many time periods. The variables that appear in these models often represent macroeconomic aggregates (such as GDP or total employment) rather than individual choice variables, and while the equations relating these variables are intended to describe economic decisions, they are not usually derived directly by aggregating models of individual choices. They are simple enough to be used as illustrations of theoretical points in introductory explanations of macroeconomic ideas; but therefore quantitative application to forecasting, testing, or policy evaluation is usually impossible without substantially augmenting the structure of the model.

Empirical forecasting models

In the 1940s and 1950s, as governments began accumulating national income and product accounting data, economists set out to construct quantitative models to describe the dynamics observed in the data.[3] These models estimated the relations between different macroeconomic variables using (mostly linear) time series analysis. Like the simpler theoretical models, these empirical models described relations between aggregate quantities, but many addressed a much finer level of detail (for example, studying the relations between output, employment, investment, and other variables in many different industries). Thus, these models grew to include hundreds or thousands of equations describing the evolution of hundreds or thousands of prices and quantities over time, making computers essential for their solution. While the choice of which variables to include in each equation was partly guided by economic theory (for example, including past income as a determinant of consumption, as suggested by the theory of adaptive expectations), variable inclusion was mostly determined on purely empirical grounds.[4]

Dutch economist Jan Tinbergen developed the first comprehensive national model, which he built for the Netherlands in 1936. He later applied the same modeling structure to the economies of the United States and the United Kingdom.[3] The first global macroeconomic model, Wharton Econometric Forecasting Associates' LINK project, was initiated by Lawrence Klein. The model was cited in 1980 when Klein, like Tinbergen before him, won the Nobel Prize. Large-scale empirical models of this type, including the Wharton model, are still in use today, especially for forecasting purposes.[5][6][7]

The Lucas critique of empirical forecasting models[edit]

Main article: Lucas critique

Econometric studies in the first part of the 20th century showed a negative correlation between inflation and unemployment called the Phillips curve.[8] Empirical macroeconomic forecasting models, being based on roughly the same data, had similar implications: they suggested that unemployment could be permanently lowered by permanently increasing inflation. However, in 1968, Milton Friedman[9] and Edmund Phelps[10] argued that this apparent tradeoff was illusory. They claimed that the historical relation between inflation and unemployment was due to the fact that past inflationary episodes had been largely unexpected. They argued that if monetary authorities permanently raised the inflation rate, workers and firms would eventually come to understand this, at which point the economy would return to its previous, higher level of unemployment, but now with higher inflation too. The stagflation of the 1970s appeared to bear out their prediction.[11]

In 1976, Robert Lucas, Jr., published an influential paper arguing that the failure of the Phillips curve in the 1970s was just one example of a general problem with empirical forecasting models.[12][13] He pointed out that such models are derived from observed relationships between various macroeconomic quantities over time, and that these relations differ depending on what macroeconomic policy regime is in place. In the context of the Phillips curve, this means that the relation between inflation and unemployment observed in an economy where inflation has usually been low in the past would differ from the relation observed in an economy where inflation has been high.[14] Furthermore, this means one cannot predict the effects of a new policy regime using an empirical forecasting model based on data from previous periods when that policy regime was not in place. Lucas argued that economists would remain unable to predict the effects of new policies unless they built models based on economic fundamentals (like preferences, technology, and budget constraints) that should be unaffected by policy changes.

Dynamic stochastic general equilibrium models

Partly as a response to the Lucas critique, economists of the 1980s and 1990s began to construct microfounded[15] macroeconomic models based on rational choice, which have come to be called **dynamic stochastic general equilibrium (DSGE)** models. These models begin by specifying the set of agents active in the economy, such as households, firms, and governments in one or more countries, as well as the preferences, technology, and budget constraint of each one. Each agent is assumed to make an optimal choice, taking into account prices and the strategies of other agents, both in the current period and in the future. Summing up the decisions of the different types of agents, it is possible to find the prices that equate supply with demand in every market. Thus these models embody a type of equilibrium self-consistency: agents choose optimally given the prices, while prices must be consistent with agents' supplies and demands.

DSGE models often assume that all agents of a given type are identical (i.e. there is a 'representative household' and a 'representative firm') and can perform perfect calculations that forecast the future correctly on average (which is called rational expectations). However, these are only simplifying assumptions, and are not essential for the DSGE methodology; many DSGE studies aim for greater realism by considering heterogeneous agents[16] or various types of adaptive expectations.[17] Compared with empirical forecasting models, DSGE models typically have fewer variables and equations, mainly because DSGE models are harder to solve, even with the help of computers.[18] Simple theoretical DSGE models, involving only a few variables, have been used to analyze the forces that drive business cycles; this empirical work has given rise to two main competing frameworks called the real business cycle model[19][20][21] and the New Keynesian DSGE model.[22][23] More elaborate DSGE models are used to predict the effects of changes in economic policy and evaluate their impact on social welfare. However, economic forecasting is still largely based on more traditional empirical models, which are still widely believed to achieve greater accuracy in predicting the impact of economic disturbances over time.

DSGE versus CGE models

A closely related methodology that pre-dates DSGE modeling is **computable general equilibrium (CGE)** modeling. Like DSGE models, CGE models are often microfounded on assumptions about preferences, technology, and budget constraints. However, CGE models focus mostly on long-run relationships, making them most suited to studying the long-run impact of permanent policies like the tax system or the openness of the economy to international trade. DSGE models instead emphasize the dynamics of the economy over time (often at a quarterly frequency), making them suited for studying business cycles and the cyclical effects of monetary and fiscal policy.

Agent-based computational macroeconomic models

Another modeling methodology which has developed at the same time as DSGE models is **Agent-based computational economics (ACE)**, which is a variety of Agent-based modeling. Like the DSGE methodology, ACE seeks to break down aggregate macroeconomic relationships into microeconomic decisions of individual agents. ACE models also begin by defining the set of agents that make up the economy, and specify the types of interactions individual agents can have with each other or with the market as a whole. Instead of defining the preferences of those agents, ACE models often jump directly to specifying their strategies. Or sometimes, preferences are specified, together with an initial strategy and a learning rule whereby the strategy is adjusted according to its past success.[27] Given these strategies, the interaction of large numbers of individual agents (who may be very heterogeneous) can be simulated on a computer, and then the aggregate, macroeconomic relationships that arise from those individual actions can be studied.

Strengths and weaknesses of DSGE and ACE models[edit]

DSGE and ACE models have different advantages and disadvantages due to their different underlying structures. DSGE models may exaggerate individual rationality and foresight, and understate the importance of heterogeneity, since the rational expectations, representative agent case remains the simplest and thus the most common type of DSGE model to solve. Also, unlike ACE models, it may be difficult to study local interactions between individual agents in DSGE models, which instead focus mostly on the way agents interact through aggregate prices. On the other hand, ACE models may exaggerate errors in individual decision-making, since the strategies assumed in ACE models may be very far from optimal choices unless the modeler is very careful. A related issue is that ACE models which start from strategies instead of preferences may remain vulnerable to the Lucas critique: a changed policy regime should generally give rise to changed strategies.

Single-equation methods (econometrics)

A variety of methods are used in econometrics to **estimate models consisting of a single equation**. The oldest and still the most commonly used is the ordinary least squares method used to estimate linear regressions.

A variety of methods are available to estimate non-linear models. A particularly important class of non-linear models are those used to estimate relationships where the dependent variable is discrete, truncated or censored. These include logit, probit and Tobit models.

Single equation methods may be applied to time-series, cross section or panel data.

Mathematical economics

Mathematical economics is the application of mathematical methods to represent theories and analyze problems in economics. By convention, these applied methods are beyond simple geometry, such as differential and integral calculus, difference and differential equations, matrix algebra, mathematical programming, and other computational methods.[1][2] Proponents of this approach claim that it allows the formulation of theoretical relationships with rigor, generality, and simplicity.[3]

Mathematics allows economists to form meaningful, testable propositions about wide-ranging and complex subjects which could less easily be expressed informally. Further, the language of mathematics allows economists to make specific, positive claims about controversial or contentious subjects that would be impossible without mathematics.[4] Much of economic theory is currently presented in terms of mathematical economic models, a set of stylized and simplified mathematical relationships asserted to clarify assumptions and implications.[5]

Broad applications include:

- optimization problems as to goal equilibrium, whether of a household, business firm, or policy maker
- static (or equilibrium) analysis in which the economic unit (such as a household) or economic system (such as a market or the economy) is modeled as not changing
- comparative statics as to a change from one equilibrium to another induced by a change in one or more factors
- dynamic analysis, tracing changes in an economic system over time, for example from economic growth.[2][6][7]

Formal economic modeling began in the 19th century with the use of differential calculus to represent and explain economic behavior, such as utility maximization, an early economic application of mathematical optimization. Economics became more mathematical as a discipline throughout the first half of the 20th century, but introduction of new and generalized techniques in the period around the Second World War, as in game theory, would greatly broaden the use of mathematical formulations in economics.[8][7]

This rapid systematizing of economics alarmed critics of the discipline as well as some noted economists. John Maynard Keynes, Robert Heilbroner, Friedrich Hayek and others have criticized the broad use of mathematical models for human behavior, arguing that some human choices are irreducible to mathematics.

Contents History

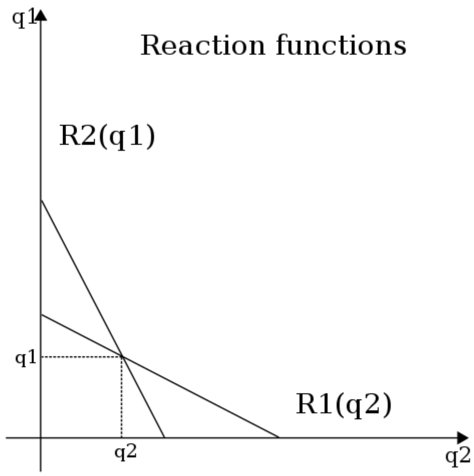
The use of mathematics in the service of social and economic analysis dates back to the 17th century. Then, mainly in German universities, a style of instruction emerged which dealt specifically with detailed presentation of data as it related to public administration. Gottfried Achenwall lectured in this fashion, coining the term statistics. At the same time, a small group of professors in England established a method of "reasoning by figures upon things relating to government" and referred to this practice as *Political Arithmetick*.[9] Sir William Petty wrote at length on issues that would later concern economists, such as taxation, Velocity of money and national income, but while his analysis was numerical, he rejected abstract mathematical methodology. Petty's use of detailed numerical data (along with John Graunt) would influence statisticians and economists for some time, even though Petty's works were largely ignored by English scholars.[10]

The mathematization of economics began in earnest in the 19th century. Most of the economic analysis of the time was what would later be called classical economics. Subjects were discussed and dispensed with through algebraic means, but calculus was not used. More importantly, until Johann Heinrich von Thünen's *The Isolated State* in 1826, economists did not develop explicit and abstract models for behavior in order to apply the tools of mathematics. Thünen's model of farmland use represents the first example of marginal analysis.[11] Thünen's work was largely theoretical, but he also mined empirical data in order to attempt to support his generalizations. In comparison to his contemporaries, Thünen built economic models and tools, rather than applying previous tools to new problems.[12]

Meanwhile, a new cohort of scholars trained in the mathematical methods of the physical sciences gravitated to economics, advocating and applying those methods to their subject,[13] and described today as moving from geometry to mechanics.[14] These included W.S. Jevons who presented paper on a "general mathematical theory of political economy" in 1862, providing an outline for use of the theory of marginal utility in political economy.[15] In 1871, he published *The Principles of Political Economy*, declaring that the subject as science "must be mathematical simply because it deals with quantities." Jevons expected that only collection of statistics for price and quantities would permit the subject as presented to become an exact science.[16] Others preceded and followed in expanding mathematical representations of economic problems.

Marginalists and the roots of neoclassical economics[edit]

Main article: Marginalism



Equilibrium quantities as a solution to two reaction functions in Cournot duopoly. Each reaction function is expressed as a linear equation dependent upon quantity demanded.

Augustin Cournot and Léon Walras built the tools of the discipline axiomatically around utility, arguing that individuals sought to maximize their utility across choices in a way that could be described mathematically.[17] At the time, it was thought that utility was quantifiable, in units known as utils.[18] Cournot, Walras and Francis Ysidro Edgeworth are considered the precursors to modern mathematical economics.[19]

Augustin Cournot[edit]

Cournot, a professor of mathematics, developed a mathematical treatment in 1838 for duopoly—a market condition defined by competition between two sellers.[19] This treatment of competition, first published in *Researches into the Mathematical Principles of Wealth*,[20] is referred to as Cournot duopoly. It is assumed that both sellers had equal access to the market and could produce their goods without cost. Further, it assumed that both goods were homogeneous. Each seller would vary her output based on the output of the other and the market price would be determined by the total quantity supplied. The profit for each firm would be determined by multiplying their output and the per unit Market price. Differentiating the profit function with respect to quantity supplied for each firm left a system of linear equations, the simultaneous solution of which gave the equilibrium quantity, price and profits.[21] Cournot's contributions to the mathematization of economics would be neglected for decades, but eventually influenced many of the marginalists.[21][22] Cournot's models of duopoly and Oligopoly also represent one of the first formulations of non-cooperative games. Today the solution can be given as a Nash equilibrium but Cournot's work preceded modern game theory by over 100 years.[23]

Léon Walras[edit]

While Cournot provided a solution for what would later be called partial equilibrium, Léon Walras attempted to formalize discussion of the economy as a whole through a theory of general competitive equilibrium. The behavior of every economic actor would be considered on both the production and consumption side. Walras originally presented four separate models of exchange, each recursively included in the next. The solution of the resulting system of equations (both linear and non-linear) is the general equilibrium.[24] At the time, no general solution could be expressed for a system of arbitrarily many equations, but Walras's attempts produced two famous results in economics. The first is Walras' law and the second is the principle of tâtonnement. Walras' method was considered highly mathematical for the time and Edgeworth commented at length about this fact in his review of *Éléments d'économie politique pure* (Elements of Pure Economics).[25]

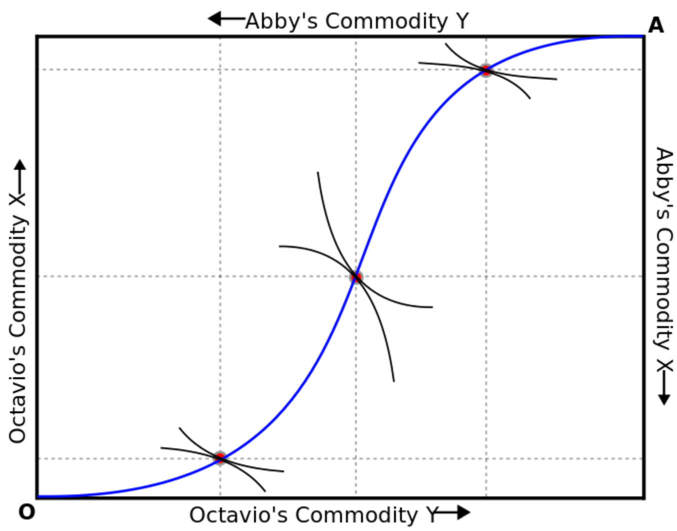
Walras' law was introduced as a theoretical answer to the problem of determining the solutions in general equilibrium. His notation is different from modern notation but can be constructed using more modern summation notation. Walras assumed that in equilibrium, all money would be spent on all goods: every good would be sold at the market price for that good and every buyer would expend their last dollar on a basket of goods. Starting from this assumption, Walras could then show that if there were n markets and n-1 markets cleared (reached equilibrium conditions) that the nth market would clear as well. This is easiest to visualize with two markets (considered in most texts as a market for goods and a market for money). If one of two markets has reached an equilibrium state, no additional goods (or conversely, money) can enter or exit the second market, so it must be in a state of equilibrium as well. Walras used this statement to move toward a proof of existence of solutions to general equilibrium but it is commonly used today to illustrate market clearing in money markets at the undergraduate level.[26]

Tâtonnement (roughly, French for *groping toward*) was meant to serve as the practical expression of Walrasian general equilibrium. Walras abstracted the marketplace as an auction of goods where the auctioneer would call out prices and market participants would wait until they could each satisfy their personal reservation prices for the quantity desired (remembering here that this is an auction on *all* goods, so everyone has a reservation price for their desired basket of goods).[27]

Only when all buyers are satisfied with the given market price would transactions occur. The market would "clear" at that price—no surplus or shortage would exist. The word *tâtonnement* is used to describe the directions the market takes in *groping toward* equilibrium, settling high or low prices on different goods until a price is agreed upon for all goods. While the process appears dynamic, Walras only presented a static model, as no transactions would occur until all markets were in equilibrium. In practice very few markets operate in this manner.[28]

Francis Ysidro Edgeworth[edit]

Edgeworth introduced mathematical elements to Economics explicitly in *Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences*, published in 1881.[29] He adopted Jeremy Bentham's felicific calculus to economic behavior, allowing the outcome of each decision to be converted into a change in utility.[30] Using this assumption, Edgeworth built a model of exchange on three assumptions: individuals are self-interested, individuals act to maximize utility, and individuals are "free to recontract with another independently of...any third party."[31]



An Edgeworth box displaying the contract curve on an economy with two participants. Referred to as the "core" of the economy in modern parlance, there are infinitely many solutions along the curve for economies with two participants[32]

Given two individuals, the set of solutions where the both individuals can maximize utility is described by the *contract curve* on what is now known as an Edgeworth Box. Technically, the construction of the two-person solution to Edgeworth's problem was not developed graphically until 1924 by Arthur Lyon Bowley.[33] The contract curve of the Edgeworth box (or more generally on any set of solutions to Edgeworth's problem for more actors) is referred to as the core of an economy.[34]

Edgeworth devoted considerable effort to insisting that mathematical proofs were appropriate for all schools of thought in economics. While at the helm of *The Economic Journal*, he published several articles criticizing the mathematical rigor of rival researchers, including Edwin Robert Anderson Seligman, a noted skeptic of mathematical economics.[35] The articles focused on a back and forth over tax incidence and responses by producers. Edgeworth noticed that a monopoly producing a good that had jointness of supply but not jointness of demand (such as first class and economy on an airplane, if the plane flies, both sets of seats fly with it) might actually lower the price seen by the consumer for one of the two commodities if a tax were applied. Common sense and more traditional, numerical analysis seemed to indicate that this was preposterous. Seligman insisted that the results Edgeworth achieved were a quirk of his mathematical formulation. He suggested that the assumption of a continuous demand function and an infinitesimal change in the tax resulted in the paradoxical predictions. Harold Hotelling later showed that Edgeworth was correct and that the same result (a "diminution of price as a result of the tax") could occur with a discontinuous demand function and large changes in the tax rate.[36]

Modern mathematical economics[edit]

From the later-1930s, an array of new mathematical tools from the differential calculus and differential equations, convex sets, and graph theory were deployed to advance economic theory in a way similar to new mathematical methods earlier applied to physics.[8][37] The process was later described as moving from mechanics to axiomatics.[38]

Differential calculus[edit]

Main articles: Foundations of Economic Analysis and Differential calculus

See also: Pareto efficiency and Walrasian auction

Vilfredo Pareto analyzed microeconomics by treating decisions by economic actors as attempts to change a given allotment of goods to another, more preferred allotment. Sets of allocations could then be treated as Pareto efficient (Pareto optimal is an equivalent term) when no exchanges could occur between actors that could make at least one individual better off without making any other individual worse off.[39] Pareto's proof is commonly conflated with Walrassian equilibrium or informally ascribed to Adam Smith's Invisible hand hypothesis.[40] Rather, Pareto's statement was the first formal assertion of what would be known as the first fundamental theorem of welfare economics.[41] These models lacked the inequalities of the next generation of mathematical economics.

In the landmark treatise *Foundations of Economic Analysis* (1947), Paul Samuelson identified a common paradigm and mathematical structure across multiple fields in the subject, building on previous work by Alfred Marshall. *Foundations* took mathematical concepts from physics and applied them to economic problems. This broad view (for example, comparing Le Chatelier's principle to tâtonnement) drives the fundamental premise of mathematical economics: systems of economic actors may be modeled and their behavior described much like any other system. This extension followed on the work of the marginalists in the previous century and extended it significantly. Samuelson approached the problems of applying individual utility maximization over aggregate groups with comparative statics, which compares two different equilibrium states after an exogenous change in a variable. This and other methods in the book provided the foundation for mathematical economics in the 20th century.[7][42]

Linear models[edit]

See also: Linear algebra, Linear programming, and Perron–Frobenius theorem

Restricted models of general equilibrium were formulated by John von Neumann in 1937.[43] Unlike earlier versions, the models of von Neumann had inequality constraints. For his model of an expanding economy, von Neumann proved the existence and uniqueness of an equilibrium using his generalization of Brouwer's fixed point theorem. Von Neumann's model of an expanding economy considered the matrix pencil $\boldsymbol{A} - \lambda \boldsymbol{B}$ with nonnegative matrices **A** and **B**; von Neumann sought probability vectors *p* and *q* and a positive number λ that would solve the complementarity equation

$$p^T (\mathbf{A} - \lambda \mathbf{B})\, q = 0,$$

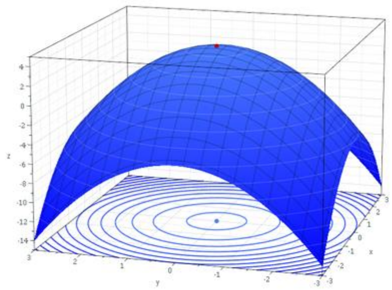
along with two inequality systems expressing economic efficiency. In this model, the (transposed) probability vector *p* represents the prices of the goods while the probability vector q represents the "intensity" at which the production process would run. The unique solution λ represents the rate of growth of the economy, which equals the interest rate. Proving the existence of a positive growth rate and proving that the growth rate equals the interest rate were remarkable achievements, even for von Neumann.[44][45][46] Von Neumann's results have been viewed as a special case of linear programming, where von Neumann's model uses only nonnegative matrices.[47] The study of von Neumann's model of an expanding economy continues to interest mathematical economists with interests in computational economics.[48][49][50]

Input-output economics[edit]

Main article: Input-output model

In 1936, the Russian–born economist Wassily Leontief built his model of input-output analysis from the 'material balance' tables constructed by Soviet economists, which themselves followed earlier work by the physiocrats. With his model, which described a system of production and demand processes, Leontief described how changes in demand in one economic sector would influence production in another.[51] In practice, Leontief estimated the coefficients of his simple models, to address economically interesting questions. In production economics, "Leontief technologies" produce outputs using constant proportions of inputs, regardless of the price of inputs, reducing the value of Leontief models for understanding economies but allowing their parameters to be estimated relatively easily. In contrast, the von Neumann model of an expanding economy allows for choice of techniques, but the coefficients must be estimated for each technology.[52][53]

Mathematical optimization[edit]



Red dot in z direction as maximum for paraboloid function of (x, y) inputs
Main articles: Mathematical optimization and Dual problem

See also: Convexity in economics and Non-convexity (economics)

In mathematics, mathematical optimization (or optimization or mathematical programming) refers to the selection of a best element from some set of available alternatives.[54] In the simplest case, an optimization problem involves maximizing or minimizing a real function by selecting input values of the function and computing the corresponding values of the function. The solution process includes satisfying general necessary and sufficient conditions for optimality. For optimization problems, specialized notation may be used as to the function and its input(s). More generally, optimization includes finding the best available element of some function given a defined domain and may use a variety of different computational optimization techniques.[55]

Economics is closely enough linked to optimization by agents in an economy that an influential definition relatedly describes economics *qua* science as the "study of human behavior as a relationship between ends and scarce means" with alternative uses.[56] Optimization problems run through modern economics, many with explicit economic or technical constraints. In microeconomics, the utility maximization problem and its dual problem, the expenditure minimization problem for a given level of utility, are economic optimization problems.[57] Theory posits that consumers maximize their utility, subject to their budget constraints and that firms maximize their profits, subject to their production functions, input costs, and market demand.[58]

Economic equilibrium is studied in optimization theory as a key ingredient of economic theorems that in principle could be tested against empirical data.[7][59] Newer developments have occurred in dynamic programming and modeling optimization with risk and uncertainty, including applications to portfolio theory, the economics of information, and search theory.[58]

Optimality properties for an entire market system may be stated in mathematical terms, as in formulation of the two fundamental theorems of welfare economics[60] and in the Arrow–Debreu model of general equilibrium (also discussed below).[61] More concretely, many problems are amenable to analytical (formulaic) solution. Many others may be sufficiently complex to require numerical methods of solution, aided by software.[55] Still others are complex but tractable enough to allow computable methods of solution, in particular computable general equilibrium models for the entire economy.[62]

Linear and nonlinear programming have profoundly affected microeconomics, which had earlier considered only equality constraints.[63] Many of the mathematical economists who received

Nobel Prizes in Economics had conducted notable research using linear programming: Leonid Kantorovich, Leonid Hurwicz, Tjalling Koopmans, Kenneth J. Arrow, Robert Dorfman, Paul Samuelson and Robert Solow.[64] Both Kantorovich and Koopmans acknowledged that George B. Dantzig deserved to share their Nobel Prize for linear programming. Economists who conducted research in nonlinear programming also have won the Nobel prize, notably Ragnar Frisch in addition to Kantorovich, Hurwicz, Koopmans, Arrow, and Samuelson.

Linear optimization[edit]

Main articles: Linear programming and Simplex algorithm

Linear programming was developed to aid the allocation of resources in firms and in industries during the 1930s in Russia and during the 1940s in the United States. During the Berlin airlift (1948), linear programming was used to plan the shipment of supplies to prevent Berlin from starving after the Soviet blockade.[65][66]

Nonlinear programming[edit]

See also: Nonlinear programming, Lagrangian multiplier, Karush–Kuhn–Tucker conditions, and Shadow price

Extensions to nonlinear optimization with inequality constraints were achieved in 1951 by Albert W. Tucker and Harold Kuhn, who considered the nonlinear optimization problem:

Minimize ( ) subject to $_i$( ) ≤ 0 and $_j$( ) $= 0$ where

(·) is the function to be minimized

$_i$(·) (= 1, ...,) are the functions of the *inequality constraints*

$_j$(·) (= 1, ...,) are the functions of the equality constraints.

In allowing inequality constraints, the Kuhn–Tucker approach generalized the classic method of Lagrange multipliers, which (until then) had allowed only equality constraints.[67] The Kuhn–Tucker approach inspired further research on Lagrangian duality, including the treatment of inequality constraints.[68][69] The duality theory of nonlinear programming is particularly satisfactory when applied to convex minimization problems, which enjoy the convex-analytic duality theory of Fenchel and Rockafellar; this convex duality is particularly strong for polyhedral convex functions, such as those arising in linear programming. Lagrangian duality and convex analysis are used daily in operations research, in the scheduling of power plants, the planning of production schedules for factories, and the routing of airlines (routes, flights, planes, crews).[69]

Variational calculus and optimal control[edit]

See also: Calculus of variations, Optimal control, and Dynamic programming

Economic dynamics allows for changes in economic variables over time, including in dynamic systems. The problem of finding optimal functions for such changes is studied in variational calculus and in optimal control theory. Before the Second World War, Frank Ramsey and Harold Hotelling used the calculus of variations to that end.

Following Richard Bellman's work on dynamic programming and the 1962 English translation of L. Pontryagin *et al*.'s earlier work,[70] optimal control theory was used more extensively in economics in addressing dynamic problems, especially as to economic growth equilibrium and stability of economic systems,[71] of which a textbook example is optimal consumption and saving.[72] A crucial distinction is between deterministic and stochastic control models.[73] Other applications of optimal control theory include those in finance, inventories, and production for example.[74]

Functional analysis[edit]

See also: Functional analysis, Convex set, Supporting hyperplane, Hahn–Banach theorem, Fixed point theorem, and Dual space

It was in the course of proving of the existence of an optimal equilibrium in his 1937 model of economic growth that John von Neumann introduced functional analytic methods to include topology in economic theory, in particular, fixed-point theory through his generalization of Brouwer's fixed-point theorem.[8][43][75] Following von Neumann's program, Kenneth Arrow and Gérard Debreu formulated abstract models of economic equilibria using convex sets and fixed–point theory. In introducing the Arrow–Debreu model in 1954, they proved the existence (but not the uniqueness) of an equilibrium and also proved that every Walras equilibrium is Pareto efficient; in general, equilibria need not be unique.[76] In their models, the ("primal") vector space represented *quantities* while the "dual" vector space represented *prices*.[77]

In Russia, the mathematician Leonid Kantorovich developed economic models in partially ordered vector spaces, that emphasized the duality between quantities and prices.[78] Kantorovich renamed *prices* as "objectively determined valuations" which were abbreviated in Russian as "o. o. o.", alluding to the difficulty of discussing prices in the Soviet Union.[77][79][80]

Even in finite dimensions, the concepts of functional analysis have illuminated economic theory, particularly in clarifying the role of prices as normal vectors to a hyperplane supporting a convex set, representing production or consumption possibilities. However, problems of describing optimization over time or under uncertainty require the use of infinite–dimensional function spaces, because agents are choosing among functions or stochastic processes.[77][81][82][83]

Differential decline and rise[edit]

See also: Global analysis, Baire category, and Sard's lemma

John von Neumann's work on functional analysis and topology broke new ground in mathematics and economic theory.[43][84] It also left advanced mathematical economics with fewer applications of differential calculus. In particular, general equilibrium theorists used general topology, convex geometry, and optimization theory more than differential calculus, because the approach of differential calculus had failed to establish the existence of an equilibrium.

However, the decline of differential calculus should not be exaggerated, because differential calculus has always been used in graduate training and in applications. Moreover, differential calculus has returned to the highest levels of mathematical economics, general equilibrium theory (GET), as practiced by the "GET-set" (the humorous designation due to Jacques H. Drèze). In the 1960s and 1970s, however, Gérard Debreu and Stephen Smale led a revival of the use of differential calculus in mathematical economics. In particular, they were able to prove the existence of a general equilibrium, where earlier writers had failed, because of their novel mathematics: Baire category from general topology and Sard's lemma from differential topology. Other economists associated with the use of differential analysis include Egbert Dierker, Andreu Mas-Colell, and Yves Balasko.[85][86] These advances have changed the traditional narrative of the history of mathematical economics, following von Neumann, which celebrated the abandonment of differential calculus.

Game theory[edit]

Main article: Game Theory

See also: Cooperative game; Noncooperative game; John von Neumann; Theory of Games and Economic Behavior; and John Forbes Nash, Jr.

John von Neumann, working with Oskar Morgenstern on the theory of games, broke new mathematical ground in 1944 by extending functional analytic methods related to convex sets and topological fixed-point theory to economic analysis.[8][84] Their work thereby avoided the traditional differential calculus, for which the maximum–operator did not apply to non-differentiable functions. Continuing von Neumann's work in cooperative game theory, game theorists Lloyd S. Shapley, Martin Shubik, Hervé Moulin, Nimrod Megiddo, Bezalel Peleg influenced economic research in politics and economics. For example, research on the fair prices in cooperative games and fair values for voting games led to changed rules for voting in legislatures and for accounting for the costs in public–works projects. For example, cooperative game theory was used in designing the water distribution system of Southern Sweden and for setting rates for dedicated telephone lines in the USA.

Earlier neoclassical theory had bounded only the *range* of bargaining outcomes and in special cases, for example bilateral monopoly or along the contract curve of the Edgeworth box.[87] Von Neumann and Morgenstern's results were similarly weak. Following von Neumann's program, however, John Nash used fixed–point theory to prove conditions under which the bargaining problem and noncooperative games can generate a unique equilibrium solution.[88] Noncooperative game theory has been adopted as a fundamental aspect of experimental economics,[89] behavioral economics,[90] information economics,[91] industrial organization,[92] and political economy.[93] It has also given rise to the subject of mechanism design (sometimes called reverse game theory), which has private and public-policy applications as to ways of improving economic efficiency through incentives for information sharing.[94]

In 1994, Nash, John Harsanyi, and Reinhard Selten received the Nobel Memorial Prize in Economic Sciences their work on non–cooperative games. Harsanyi and Selten were awarded for their work on repeated games. Later work extended their results to computational methods of modeling.[95]

Agent-based computational economics[edit]

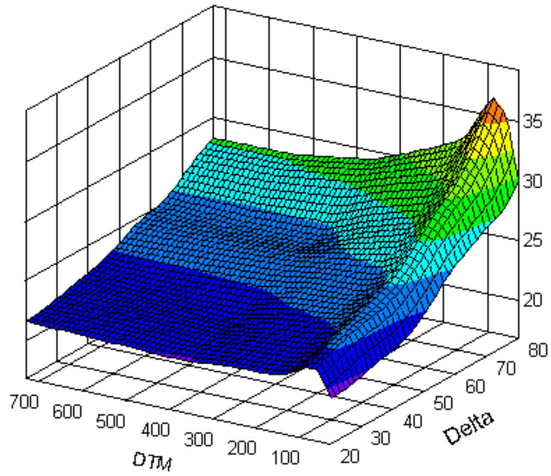
Main article: Agent-based computational economics

Agent-based computational economics (ACE) as a named field is relatively recent, dating from about the 1990s as to published work. It studies economic processes, including whole economies, as dynamic systems of interacting agents over time. As such, it falls in the paradigm of complex adaptive systems.[96] In corresponding agent-based models, agents are not real people but "computational objects modeled as interacting according to rules" ... "whose micro-level interactions create emergent patterns" in space and time.[97] The rules are formulated to predict behavior and social interactions based on incentives and information. The theoretical assumption of mathematical *optimization* by agents markets is replaced by the less restrictive postulate of agents with *bounded* rationality *adapting* to market forces.[98]

ACE models apply numerical methods of analysis to computer-based simulations of complex dynamic problems for which more conventional methods, such as theorem formulation, may not find ready use.[99] Starting from specified initial conditions, the computational economic system is modeled as evolving over time as its constituent agents repeatedly interact with each other. In these respects, ACE has been characterized as a bottom-up culture-dish approach to the study of the economy.[100] In contrast to other standard modeling methods, ACE events are driven solely by initial conditions, whether or not equilibria exist or are computationally tractable. ACE modeling, however, includes agent adaptation, autonomy, and learning.[101] It has a similarity to, and overlap with, game theory as an agent-based method for modeling social interactions.[95] Other dimensions of the approach include such standard economic subjects as competition and collaboration,[102] market structure and industrial organization,[103] transaction costs,[104] welfare economics[105] and mechanism design,[94] information and uncertainty,[106] and macroeconomics.[107][108]

The method is said to benefit from continuing improvements in modeling techniques of computer science and increased computer capabilities. Issues include those common to experimental economics in general[109] and by comparison[110] and to development of a common framework for empirical validation and resolving open questions in agent-based modeling.[111] The ultimate scientific objective of the method has been described as "test[ing] theoretical findings against real-world data in ways that permit empirically supported theories to cumulate over time, with each researcher's work building appropriately on the work that has gone before."[112]

Mathematicization of economics[edit]



The surface of the Volatility smile is a 3-D surface whereby the current market implied volatility (Z-axis) for all options on the underlier is plotted against strike price and time to maturity (X & Y-axes).[113]

Over the course of the 20th century, articles in "core journals"[114] in economics have been almost exclusively written by economists in academia. As a result, much of the material transmitted in those journals relates to economic theory, and "economic theory itself has been continuously more abstract and mathematical."[115] A subjective assessment of mathematical techniques[116] employed in these core journals showed a decrease in articles that use neither geometric representations nor mathematical notation from 95% in 1892 to 5.3% in 1990.[117] A 2007 survey of ten of the top economic journals finds that only 5.8% of the articles published in 2003 and 2004 both lacked statistical analysis of data and lacked displayed mathematical expressions that were indexed with numbers at the margin of the page.[118]

Econometrics[edit]

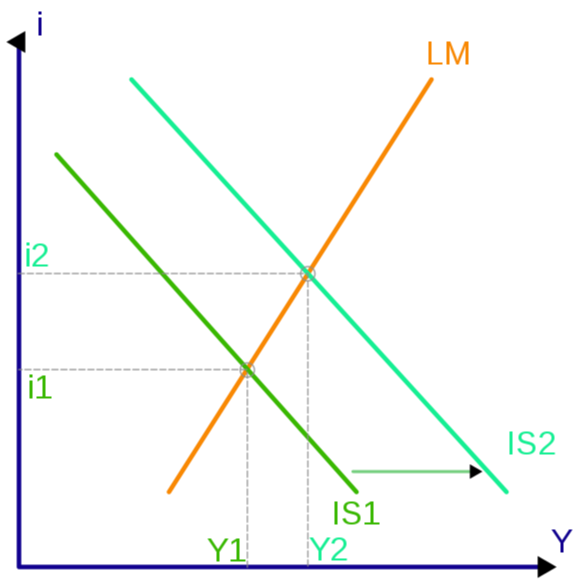
Main article: Econometrics

Between the world wars, advances in mathematical statistics and a cadre of mathematically trained economists led to econometrics, which was the name proposed for the discipline of advancing economics by using mathematics and statistics. Within economics, "econometrics" has often been used for statistical methods in economics, rather than mathematical economics. Statistical econometrics features the application of linear regression and time series analysis to economic data.

Ragnar Frisch coined the word "econometrics" and helped to found both the Econometric Society in 1930 and the journal *Econometrica* in 1933.[119][120] A student of Frisch's, Trygve Haavelmo published *The Probability Approach in Econometrics* in 1944, where he asserted that precise statistical analysis could be used as a tool to validate mathematical theories about economic actors with data from complex sources.[121] This linking of statistical analysis of systems to economic theory was also promulgated by the Cowles Commission (now the Cowles Foundation) throughout the 1930s and 1940s.[122]

The roots of modern econometrics can be traced to the American economist Henry L. Moore. Moore studied agricultural productivity and attempted to fit changing values of productivity for plots of corn and other crops to a curve using different values of elasticity. Moore made several errors in his work, some from his choice of models and some from limitations in his use of mathematics. The accuracy of Moore's models also was limited by the poor data for national accounts in the United States at the time. While his first models of production were static, in 1925 he published a dynamic "moving equilibrium" model designed to explain business cycles—this periodic variation from over-correction in supply and demand curves is now known as the cobweb model. A more formal derivation of this model was made later by Nicholas Kaldor, who is largely credited for its exposition.[123]

Application[edit]



The IS/LM model is a Keynesian macroeconomic model designed to make predictions about the intersection of "real" economic activity (e.g. spending, income, savings rates) and decisions made in the financial markets (Money supply and Liquidity preference). The model is no longer widely taught at the graduate level but is common in undergraduate macroeconomics courses.[124]

Much of classical economics can be presented in simple geometric terms or elementary mathematical notation. Mathematical economics, however, conventionally makes use of calculus and matrix algebra in economic analysis in order to make powerful claims that would be more difficult without such mathematical tools. These tools are prerequisites for formal study, not only in mathematical economics but in contemporary economic theory in general. Economic problems often involve so many variables that mathematics is the only practical way of attacking and solving them. Alfred Marshall argued that every economic problem which can be quantified, analytically expressed and solved, should be treated by means of mathematical work.[125]

Economics has become increasingly dependent upon mathematical methods and the mathematical tools it employs have become more sophisticated. As a result, mathematics has become considerably more important to professionals in economics and finance. Graduate programs in both economics and finance require strong undergraduate preparation in mathematics for admission and, for this reason, attract an increasingly high number of mathematicians. Applied mathematicians apply mathematical principles to practical problems, such as economic analysis and other economics-related issues, and many economic problems are often defined as integrated into the scope of applied mathematics.[17]

This integration results from the formulation of economic problems as stylized models with clear assumptions and falsifiable predictions. This modeling may be informal or prosaic, as it was in Adam Smith's *The Wealth of Nations*, or it may be formal, rigorous and mathematical.

Broadly speaking, formal economic models may be classified as stochastic or deterministic and as discrete or continuous. At a practical level, quantitative modeling is applied to many areas of economics and several methodologies have evolved more or less independently of each other.[126]

- Stochastic models are formulated using stochastic processes. They model economically observable values over time. Most of econometrics is based on statistics to formulate and test hypotheses about these processes or estimate parameters for them. Between the World Wars, Herman Wold developed a representation of stationary stochastic processes in terms of autoregressive models and a determinist trend. Wold and Jan Tinbergen applied time-series analysis to economic data. Contemporary research on time series statistics consider additional formulations of stationary processes, such as autoregressive moving average models. More general models include autoregressive conditional heteroskedasticity (ARCH) models and generalized ARCH (GARCH) models.
- Non-stochastic mathematical models may be purely qualitative (for example, models involved in some aspect of social choice theory) or quantitative (involving rationalization of financial variables, for example with hyperbolic coordinates, and/or specific forms of functional relationships between variables). In some cases economic predictions of a model merely assert the direction of movement of economic variables, and so the functional relationships are used only in a qualitative sense: for example, if the price of an item increases, then the demand for that item will decrease. For such models, economists often use two-dimensional graphs instead of functions.
- Qualitative models are occasionally used. One example is qualitative scenario planning in which possible future events are played out. Another example is non-numerical decision tree analysis. Qualitative models often suffer from lack of precision.

Example: The effect of a corporate tax cut on wages[edit]

The great appeal of mathematical economics is that it brings a degree of rigor to economic thinking, particularly around charged political topics. For example, during the discussion of the efficacy of a corporate tax cut for increasing the wages of workers, a simple mathematical model proved beneficial to understanding the issues at hand.

As an intellectual exercise, the following problem was posed by Prof. Greg Mankiw of Harvard University:[127]

An open economy has the production function , where is output per worker and is capital per worker. The capital stock adjusts so that the after-tax marginal product of capital equals the exogenously given world interest rate ...How much will the tax cut increase wages?

To answer this question, we follow John H. Cochrane of the Hoover Institution.[128] Suppose an open economy has the production function:

Where the variables in this equation are:

- is the total output
- is the production function
- is the total capital stock
- is the total labor stock

The standard choice for the production function is the Cobb-Douglas production function:

where is the factor of productivity - assumed to be a constant. A corporate tax cut in this model is equivalent to a tax on capital. With taxes, firms look to maximize:

where is the capital tax rate, is wages per worker, and is the exogenous interest rate. Then the first-order optimality conditions become:

Therefore, the optimality conditions imply that:

Define total taxes . This implies that taxes per worker are:

Then the change in taxes per worker, given the tax rate, is:

To find the change in wages, we differentiate the second optimality condition for the per worker wages to obtain:

Assuming that the interest rate is fixed at , so that , we may differentiate the first optimality condition for the interest rate to find:

For the moment, let's focus only on the **static** effect of a capital tax cut, so that . If we substitute this equation into equation for wage changes with respect to the tax rate, then we find that:

Therefore, the **static** effect of a capital tax cut on wages is:

Based on the model, it seems possible that we may achieve a rise in the wage of a worker greater than the amount of the tax cut. But that only considers the static effect, and we know that the dynamic effect must be accounted for. In the dynamic model, we may rewrite the equation for changes in taxes per worker with respect to the tax rate as:

Recalling that , we have that:

Using the Cobb-Douglas production function, we have that:

Therefore, the **dynamic** effect of a capital tax cut on wages is:

If we take , then the dynamic effect of lowering capital taxes on wages will be even larger than the static effect. Moreover, if there are positive externalities to capital accumulation, the effect of the tax cut on wages would be larger than in the model we just derived. It is important to note that the result is a combination of:

1. The standard result that in a small open economy labor bears 100% of a small capital income tax
2. The fact that, starting at a positive tax rate, the burden of a tax increase exceeds revenue collection due to the first-order deadweight loss

This result showing that, under certain assumptions, a corporate tax cut can boost the wages of workers by more than the lost revenue does not imply that the magnitude is correct. Rather, it suggests a basis for policy analysis that is not grounded in handwaving. If the assumptions are reasonable, then the model is an acceptable approximation of reality; if they are not, then better models should be developed.

CES production function[edit]

Now let's assume that instead of the Cobb-Douglas production function we have a more general constant elasticity of substitution (CES) production function:

where ; is the elasticity of substitution between capital and labor. The relevant quantity we want to calculate is , which may be derived as:

Therefore, we may use this to find that:

Therefore, under a general CES model, the dynamic effect of a capital tax cut on wages is:

We recover the Cobb-Douglas solution when . When , which is the case when perfect substitutes exist, we find that - there is no effect of changes in capital taxes on wages. And when , which is the case when perfect complements exist, we find that - a cut in capital taxes increases wages by exactly one dollar.

Classification[edit]

According to the Mathematics Subject Classification (MSC), mathematical economics falls into the Applied mathematics/other classification of category 91:

> Game theory, economics, social and behavioral sciences

with MSC2010 classifications for 'Game theory' at codes 91Axx and for 'Mathematical economics' at codes 91Bxx.

The *Handbook of Mathematical Economics* series (Elsevier), currently 4 volumes, distinguishes between *mathematical methods in economics*, v. 1, Part I, and *areas of economics* in other volumes where mathematics is employed.[129]

Another source with a similar distinction is *The New Palgrave: A Dictionary of Economics* (1987, 4 vols., 1,300 subject entries). In it, a "Subject Index" includes mathematical entries under 2 headings (vol. IV, pp. 982–3):

Mathematical Economics (24 listed, such as "acyclicity", "aggregation problem", "comparative statics", "lexicographic orderings", "linear models", "orderings", and "qualitative economics")
Mathematical Methods (42 listed, such as "calculus of variations", "catastrophe theory", "combinatorics," "computation of general equilibrium", "convexity", "convex programming", and "stochastic optimal control").

A widely used system in economics that includes mathematical methods on the subject is the JEL classification codes. It originated in the *Journal of Economic Literature* for classifying new books and articles. The relevant categories are listed below (simplified below to omit "Miscellaneous" and "Other" JEL codes), as reproduced from JEL classification codes#Mathematical and quantitative methods JEL: C Subcategories. *The New Palgrave Dictionary of Economics* (2008, 2nd ed.) also uses the JEL codes to classify its entries. The corresponding footnotes below have links to abstracts of *The New Palgrave Online* for each JEL category (10 or fewer per page, similar to Google searches).

JEL: C02 - Mathematical Methods (following JEL: C00 - General and JEL: C01 - Econometrics)
JEL: C6 - Mathematical Methods; Programming Models; Mathematical and Simulation Modeling [130]
JEL: C60 - General
JEL: C61 - Optimization techniques; Programming models; Dynamic analysis[131]
JEL: C62 - Existence and stability conditions of equilibrium[132]
JEL: C63 - Computational techniques; Simulation modeling[133]
JEL: C67 - Input–output models
JEL: C68 - Computable General Equilibrium models[134]
JEL: C7 - Game theory and Bargaining theory[135]
JEL: C70 - General[136]
JEL: C71 - Cooperative games[137]
JEL: C72 - Noncooperative games[138]
JEL: C73 - Stochastic and Dynamic games; Evolutionary games; Repeated Games[139]
JEL: C78 - Bargaining theory; Matching theory[140]

Criticisms and defences[edit]

Adequacy of mathematics for qualitative and complicated economics[edit]

Friedrich Hayek contended that the use of formal techniques projects a scientific exactness that does not appropriately account for informational limitations faced by real economic agents. [141]

In an interview in 1999, the economic historian Robert Heilbroner stated:[142]

I guess the scientific approach began to penetrate and soon dominate the profession in the past twenty to thirty years. This came about in part because of the "invention" of mathematical analysis of various kinds and, indeed, considerable improvements in it. This is the age in which we have not only more data but more sophisticated use of data. So there is a strong feeling that this is a data-laden science and a data-laden undertaking, which, by virtue of the sheer numerics, the sheer equations, and the sheer look of a journal page, bears a certain resemblance to science . . . That one central activity looks scientific. I understand that. I think that is genuine. It approaches being a universal law. But resembling a science is different from being a science.

Heilbroner stated that "some/much of economics is not naturally quantitative and therefore does not lend itself to mathematical exposition."[143]

Testing predictions of mathematical economics[edit]

Philosopher Karl Popper discussed the scientific standing of economics in the 1940s and 1950s. He argued that mathematical economics suffered from being tautological. In other words, insofar as economics became a mathematical theory, mathematical economics ceased to rely on empirical refutation but rather relied on mathematical proofs and disproof.[144] According to Popper, falsifiable assumptions can be tested by experiment and observation while unfalsifiable assumptions can be explored mathematically for their consequences and for their consistency with other assumptions.[145]

Sharing Popper's concerns about assumptions in economics generally, and not just mathematical economics, Milton Friedman declared that "all assumptions are unrealistic". Friedman proposed judging economic models by their predictive performance rather than by the match between their assumptions and reality.[146]

Mathematical economics as a form of pure mathematics

Considering mathematical economics, J.M. Keynes wrote in *The General Theory*:[147]

It is a great fault of symbolic pseudo-mathematical methods of formalising a system of economic analysis ... that they expressly assume strict independence between the factors involved and lose their cogency and authority if this hypothesis is disallowed; whereas, in ordinary discourse, where we are not blindly manipulating and know all the time what we are doing and what the words mean, we can keep ‘at the back of our heads’ the necessary reserves and qualifications and the adjustments which we shall have to make later on, in a way in which we cannot keep complicated partial differentials ‘at the back’ of several pages of algebra which assume they all vanish. Too large a proportion of recent ‘mathematical’ economics are merely concoctions, as imprecise as the initial assumptions they rest on, which allow the author to lose sight of the complexities and interdependencies of the real world in a maze of pretentious and unhelpful symbols.

Defense of mathematical economics]

In response to these criticisms, Paul Samuelson argued that mathematics is a language, repeating a thesis of Josiah Willard Gibbs. In economics, the language of mathematics is sometimes necessary for representing substantive problems. Moreover, mathematical economics has led to conceptual advances in economics.[148] In particular, Samuelson gave the example of microeconomics, writing that "few people are ingenious enough to grasp [its] more complex parts... *without* resorting to the language of mathematics, while most ordinary individuals can do so fairly easily *with* the aid of mathematics."[149]

Some economists state that mathematical economics deserves support just like other forms of mathematics, particularly its neighbors in mathematical optimization and mathematical statistics and increasingly in theoretical computer science. Mathematical economics and other mathematical sciences have a history in which theoretical advances have regularly contributed to the reform of the more applied branches of economics. In particular, following the program of John von Neumann, game theory now provides the foundations for describing much of applied economics, from statistical decision theory (as "games against nature") and econometrics to general equilibrium theory and industrial organization. In the last decade, with the rise of the internet, mathematical economists and optimization experts and computer scientists have worked on problems of pricing for on-line services --- their contributions using mathematics from cooperative game theory, nondifferentiable optimization, and combinatorial games.

Robert M. Solow concluded that mathematical economics was the core "infrastructure" of contemporary economics:

Economics is no longer a fit conversation piece for ladies and gentlemen. It has become a technical subject. Like any technical subject it attracts some people who are more interested in the technique than the subject. That is too bad, but it may be inevitable. In any case, do not kid yourself: the technical core of economics is indispensable infrastructure for the political economy. That is why, if you consult [a reference in contemporary economics] looking for enlightenment about the world today, you will be led to technical economics, or history, or nothing at all.[150]

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